Cameo Wood 03/06/2003

History of Mathematics – Assignment 1
Early Mathematics

1. Is the Chinese number system a simple grouping system or a positional system or neither? Explain. Write 367 and 6024 in the Chinese, Egyptian and Babylonian number system.

The Chinese number system is a positional system because the order is critical in understanding how to calculate the appropriate amounts. While you could express the number 367 as '7, 6, 10, 3, 100' and still have it be comprehensible, you could not represent the numbers in a random order; But you could rearrange them in pairs, using the 10,100,1000 symbols appropriately. Therefore, it is positional. 367 is $(3 \cdot 100) + (6 \cdot 10) + 7$. 6024 is $(6 \cdot 1000) + (2 \cdot 10) + 4$.

Chinese

367:

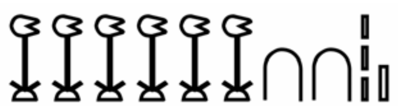
三
百
六
十
七

6024:

六
千
二
十
四

Egyptian

(3x100) + (6x10) + 7 = 367



(6x1000) + (2x10) + 4 = 6024

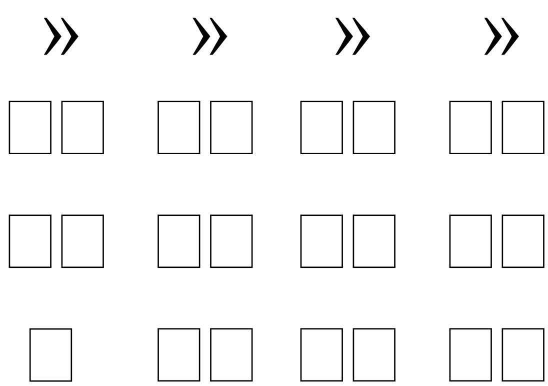
Babylonian

(6x60) + 7 = 367

(3600x1) +(60x40)+24=6024

2. Egyptian number system:
   a. Write $\frac{5}{9}$ in the Egyptian number system following the two rules: You can only use unit fractions and you cannot use the same fraction twice.

$$\frac{5}{9} = \frac{2}{9}+\frac{2}{9}+\frac{1}{9}$$
$$= \frac{1}{6}+\frac{1}{18}+\frac{1}{6}+\frac{1}{18}+\frac{1}{9}$$
$$= \frac{1}{3}+\frac{2}{9}$$
$$\boxed{= \frac{1}{3}+\frac{1}{6}+\frac{1}{18}}$$



b. The pattern from entries with denominators that are multiples of 3 ($\frac{2}{9}, \frac{2}{15}, \frac{2}{21}$ etc.) can be used to determine the entries in the table for $\frac{2}{39}$ and $\frac{2}{45}$. Therefore, $\frac{2}{39} = \frac{1}{26} + \frac{1}{78}$ and $\frac{2}{45} = \frac{1}{30} + \frac{1}{90}$ using the formula $\frac{2}{3n} = \frac{1}{2n} + \frac{1}{6n}$.

	A		B	C
1	$\frac{2}{9}$	=	$\frac{1}{6}$	$\frac{1}{18}$
2	$\frac{2}{15}$	=	$\frac{1}{10}$	$\frac{1}{30}$
3	$\frac{2}{21}$	=	$\frac{1}{14}$	$\frac{1}{42}$
4	$\frac{2}{27}$	=	$\frac{1}{18}$	$\frac{1}{54}$
5	$\frac{2}{33}$	=	$\frac{1}{22}$	$\frac{1}{66}$
6	$\frac{2}{39}$	=	$\frac{1}{26}$	$\frac{1}{78}$
7	$\frac{2}{45}$	=	$\frac{1}{30}$	$\frac{1}{90}$

I. The denominator of the fraction in column A increases by 6 as you move down the table. (i.e. the denominator of the fraction in A2 is 6 more than the denominator in A1.)

II. The denominator of the fraction in column B increases by 4 as you move down the table. (i.e. the denominator of the fraction in B2 is 4 more than the denominator in B1.)

III. The denominator of the fraction in column C increases by 12 as you move down the table. (i.e. the denominator of the fraction in C2 is 12 more than the denominator in C1.)

IV. The denominator of the fraction in the first column of a row is increasing by a series of odd numbers (i.e. 3,5,7,9,11, etc.) to produce the denominator number in the second column, while the denominator number in the third column of a row is produced by doubling the first denominator.

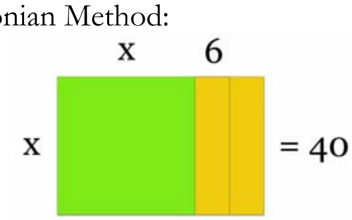
c. The fractions $\frac{2}{3n}$ are written as the sum of two fractions, $\frac{1}{2n}$ and $\frac{1}{6n}$.

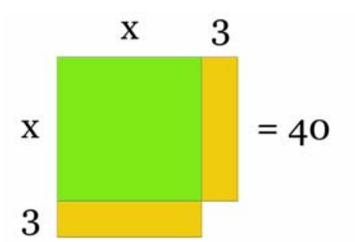
$$\frac{2}{39} = \frac{1}{26} + \frac{1}{78} \equiv \frac{2}{3 \cdot 13} = \frac{1}{2 \cdot 13} + \frac{1}{6 \cdot 13}$$
$$\frac{2}{45} = \frac{1}{30} + \frac{1}{90} \equiv \frac{2}{3 \cdot 15} = \frac{1}{2 \cdot 15} + \frac{1}{6 \cdot 15}$$

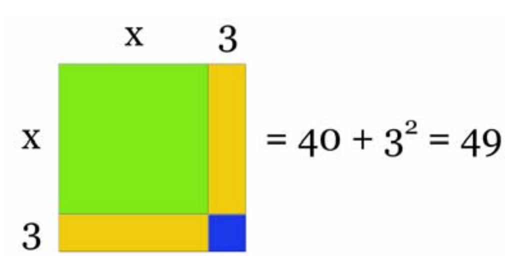
3. Babylonian Quadratics:

a. "I have added the area and six sides to my square and it is 40." Translation $\boxed{x + 6x = 40}$.

i. Babylonian Method:

1. 

2. 

3. 

4. Area of whole completed square (S^2) = 49; therefore, the area of each side of the completed square (S) = 7

5. 7= x+3; therefore, x=4

ii. Our Method:

1. $x^2 + 6x = 40$
2. $x^2 + 6x - 40 = 0$
3. Use the Quadratic equation
4. Answer: $x = 4$

b. The equation that the student is attempting to solve is 3x + 5x = 32. The student's proposed method involves adding the two sides of the square together (i.e. 3x + 5x to yield 8x) and then dividing 32 by this number, 8, to yield 4. This method appears correct, since it is essentially the same method that we would use to obtain a value for x (i.e. we would go through similar motions algebraically: 3x + 5x = 32; 8x = 32; x=4)

4. Translate the Babylonian tablet into our number system. Explain the pattern of numbers on the tablet. Using your answer to a., calculate the next six rows of the table. Use your equation to find a solution to $x^3 + x^2 = 576$.

x	$x^3 + x^2 =$
1	2
2	12
3	36
4	80

x	x^3	x^2	$x^3 + x^2 =$
5	125	25	150
6	216	36	252
7	343	49	392
8	512	64	576
9	729	81	810
10	1000	100	1100

$x^3 + x^2 = 576$

$8^3 + 8^2 = 576$

5. Determine the value of π used by the Hebrews from this quote from I Kings 7:23 (and II Chronicles 4:2):

"He made the Sea of cast metal, circular in shape, measuring ten cubits from rim to rim and five cubits high. It took a line of thirty cubits to measure around it."

c=30 d=10 $30 \div 10 = 3$ **$\pi = 3$**