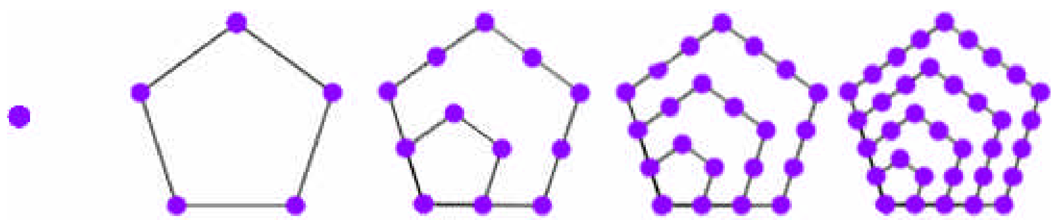
Wood, C.
Assignment 2 – Greek Mathematics
History of Mathematics
Professor Van Brummelen
April 16, 2003

1. Pentagonal Numbers:

   a. First five pentagonal numbers:



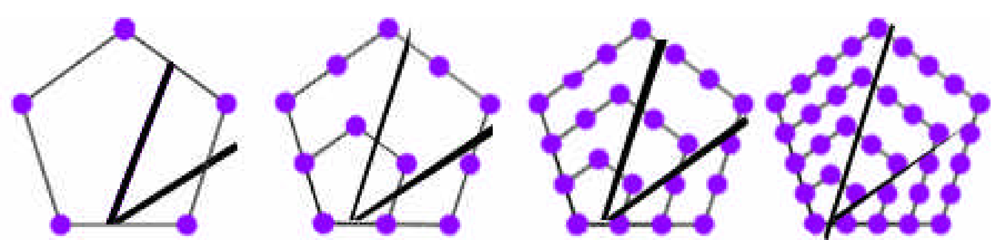
1 **5** **12** **22** **35**

   b. Relation between triangular and pentagonal numbers:

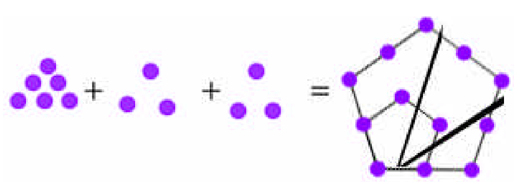
Pentagonal	1	5	12	22	35
Triangular	1	3	6	10	15

A pentagonal number of 5 consists of three triangular numbers, 1 and 3. The formula to make a triangular number is to add the triangular number directly below it in the above chart, and then to add the number to the left of that triangular number twice. So, the pentagonal number 5 is thus: 1+1+3=5. The pentagonal number 22: 6+6+10=22.

   c. Illustrate that a pentagonal number is the sum of three triangles:



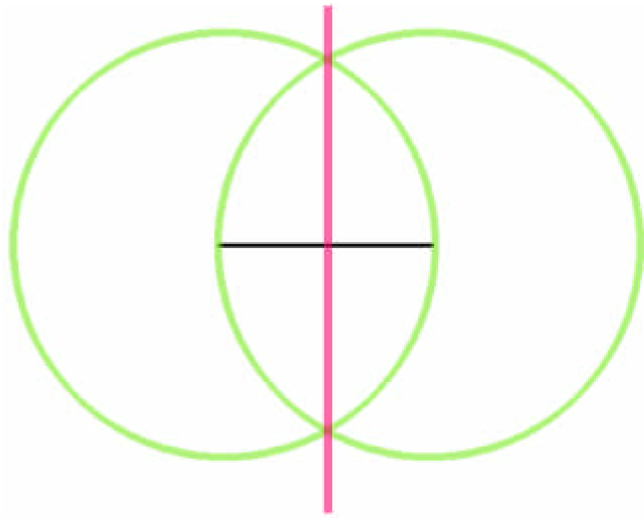
1+1+3=5 **3+3+6=12** **6+6+10=22** **10+10+15=35**



6 + 3 + 3 = 12

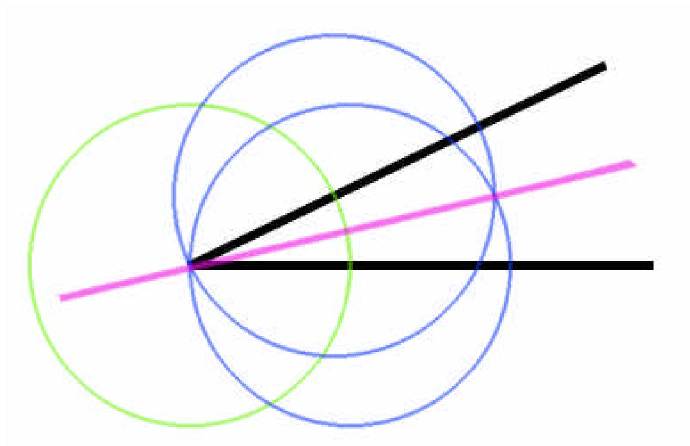
2. Euclidean tools:

a. Bisect a line segment:



First a line of any length is drawn. Then, two green circles are drawn using the entire length of the line as the radius from each end of the line. A pink line intersects where the two circles intersect to find the exact middle of the line. This bisection is exactly half because both circles are of equal length and follow Euclid's Common Notions I and II.

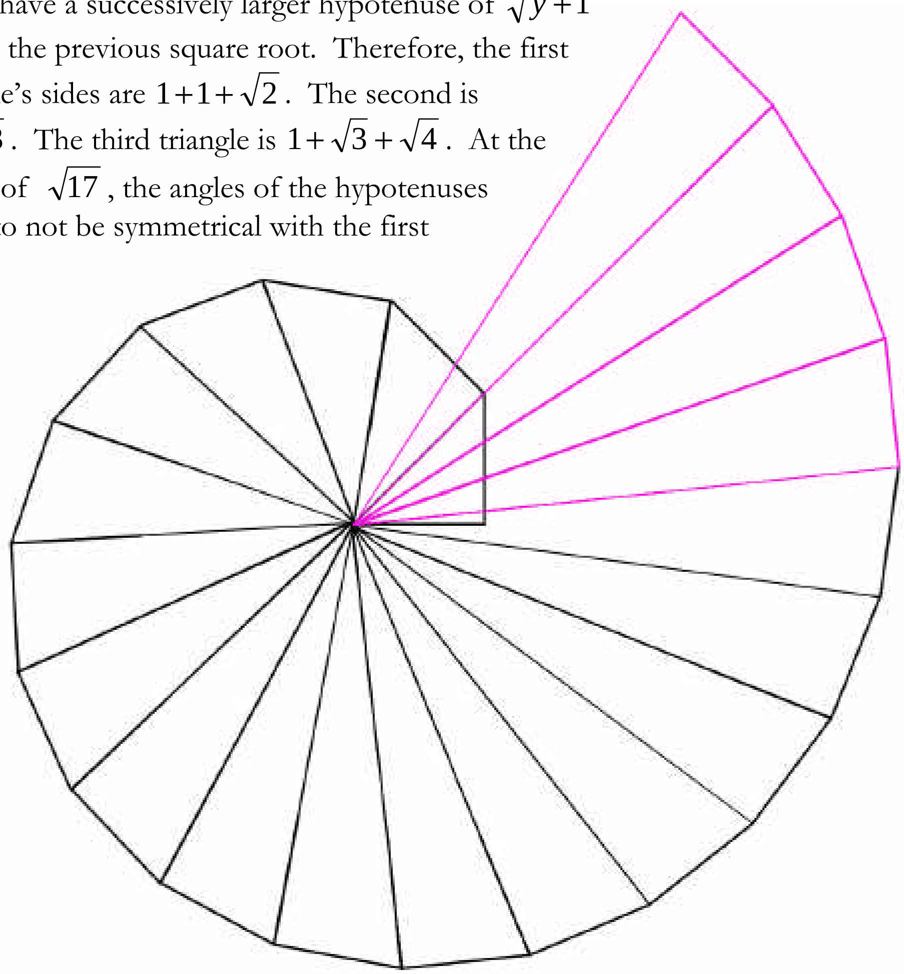
b. Bisect an angle



An angle of any size is drawn, and then a green circle of any size is drawn with its center the intersection of points of the angle. Two (blue) circles are drawn with the center on the line of the angle where it intersects with the circle that was just drawn. The Radius is from the intersection point of the circle and one of the lines of the angle to the angle intersection point. Another blue circle is drawn on the other line of the angle in the same manner. The bisection is found in pink when one draws a line from the angle intersection point through where the two blue circles intersect.

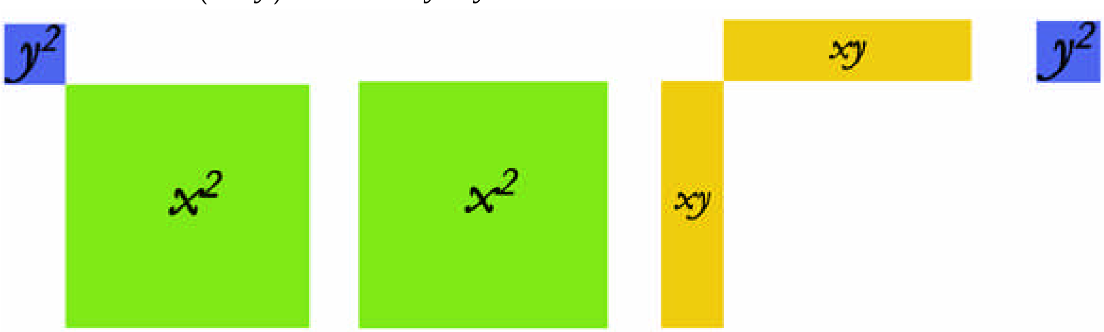
3. Plato's dialogue <u>Theætetus</u> and the discovery that $\sqrt{2}$ is irrational.

The first triangle is a right angled isosceles triangle with two sides that have a length of 1 and a hypotenuse of $\sqrt{2}$. As the right-angled triangles continue, they always have a successively larger hypotenuse of $\sqrt{y+1}$ with *y* being the previous square root. Therefore, the first small triangle's sides are $1+1+\sqrt{2}$. The second is $1+\sqrt{2}+\sqrt{3}$. The third triangle is $1+\sqrt{3}+\sqrt{4}$. At the hypotenuse of $\sqrt{17}$, the angles of the hypotenuses are proven to not be symmetrical with the first triangle..



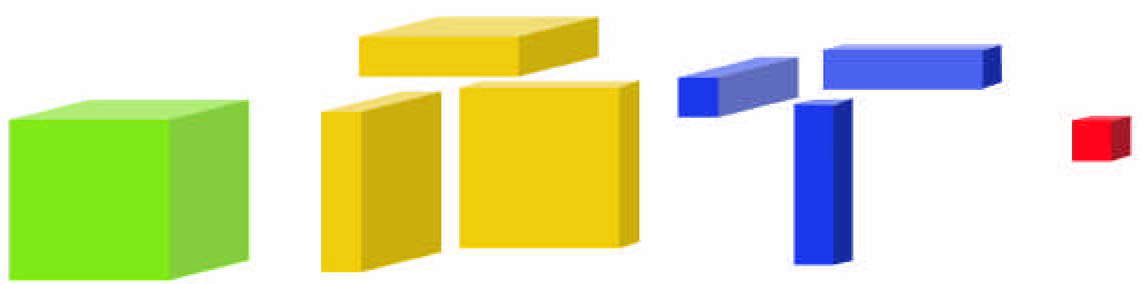
4. Greek geometrical algebra:

a. $(x-y)^2 = x^2 - 2xy + y^2$

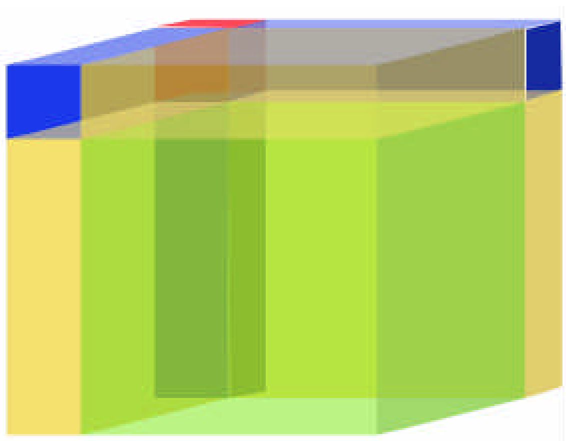


$(x-y)^2 \quad = \quad x^2 \quad - 2xy \quad y^2$

b. $(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$



x^3 $3x^2y$ $3xy^2$ y^3



$(x+y)^3$

5. Ptolemy and sines:

a. Value of the sine of 45°

$$\sin = \frac{x}{1}$$

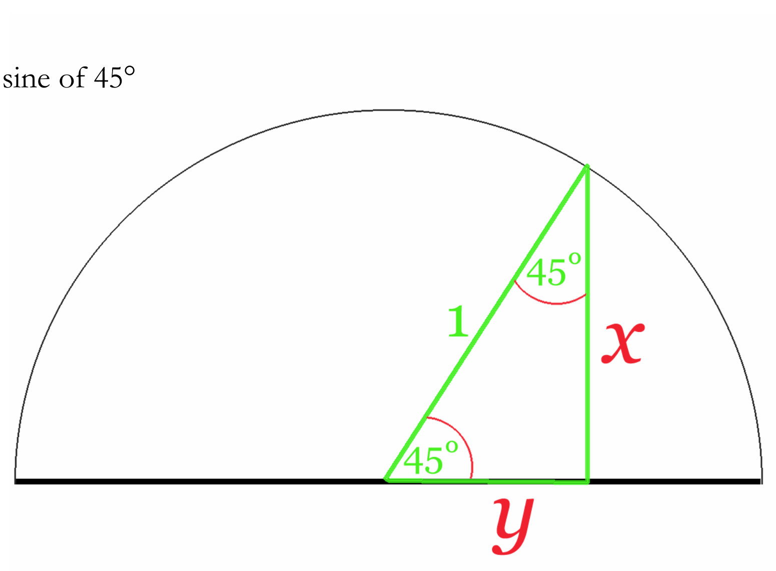
$$x^2 + x^2 = 1^2$$

$$\frac{2x^2}{2} = \frac{1^2}{2}$$

$$x^2 = \frac{1}{2}$$

$$\sqrt{x^2} = \sqrt{\frac{1}{2}}$$

$$x = \sqrt{\frac{1}{2}}$$



b. With only the starting sines, what is the smallest whole numbered sine?

Using the starting sines 18°, 30°, 36° and 45°, I used the second tool of finding the sum of two signs, and found that 18° +30° = 48°. Then, I used the third rule to find the difference, 48°-45° = 3°.

c. Ptolemy and approximate method:

Sine of $\frac{3}{4}° = 0.013091$

Sine of $1\frac{1}{2}° = 0.026172$

$$\frac{\sin a}{\sin b} < \frac{a}{b}$$

$$\frac{\sin 1°}{\sin .013091°} < \frac{1°}{.75°}$$

$$\frac{\sin 1°}{\sin .013091°} < 1.333\ldots°$$

$$\sin 1° < 0.174546°$$

$$\frac{\sin 1°}{\sin .026172°} > \frac{1°}{1.5°}$$

$$\frac{\sin 1°}{\sin .026172°} > .66667°$$

$$\sin 1° > 0.017448°$$

$$\boxed{0.174546 < \sin 1° > 0.017448}$$

And finally, a joke for you:

After explaining to a student, with various lessons and examples, that:

$$\lim_{x \to 8} \frac{1}{x-8} = \infty$$

I tried to check if she really understood that, so I gave her a different example.

This was the result:

$$\lim_{x \to 5} \frac{1}{x-5} = \rotatebox{90}{5}$$